

Preface

WHY THIS BOOK NOW?

This book is a comprehensive guide to fostering and supporting mathematical argumentation among your middle school students. We offer it to all teachers who are currently grappling with how to increase and strengthen mathematical discourse in their classrooms. It is designed to be a practical guidebook, with advice on teaching, activity handouts, links to interactive web pages that go with the activities, warm-up games, and lesson planning ideas.

Mathematical argumentation is important not only because it appears in current state standards but, more fundamentally, because it is also what mathematicians do. All students deserve to have access to this high-level disciplinary practice. Additionally, the principles and logic of mathematical argumentation are very relevant to the kinds of reasoning your students need to be involved citizens and to have successful careers.

Experience with mathematical argumentation is critical for students in middle grades. For too long, argumentation has been associated with two-column geometric proofs, taught by rote in high school. While numerous studies have documented high school and even college students' troubles with proof (e.g., Healy & Hoyles, 2000), there are also promising studies of even elementary school students learning to make generalized arguments (e.g., Ball & Bass, 2000; Schifter, 2009). Middle school is a crucial transition period when students refine their understanding of generalization by paying attention to patterns and move beyond the consideration of individual cases.

All teaching is complex work, and teaching for argumentation is even more so. We hope to honor your complex work by portraying it and guiding it as “disciplined improvisation” (Sawyer, 2011). We help you develop specific teaching moves you can call on when you have to think on your feet as a surprising argument unfolds in class. Teaching moves are behaviors that are aimed at a purpose: For example, we provide both specific questions you can ask and the reasons for asking them.

HOW THIS BOOK WORKS

The book has eight chapters.

Chapter 1 is an introduction to mathematical argumentation and why you should teach it.

Chapters 2, 3, 4, and 7 are each devoted to one part of our model of argumentation—generating cases, conjecturing, justifying, and concluding.

Justifying is so important that we devote two more chapters to it: Chapter 5 on the use of representations and Chapter 6 about the levels of justifications your students will make.

The final chapter, Chapter 8, is about how to plan to put together all the pieces presented in the earlier chapters.

Most chapters have at least one vignette from the classroom. These vignettes are sometimes based on what we have observed in the classrooms of teachers who were participating in the Bridging Professional Development program that we work on at SRI International, but they also portray a composite of several teachers and our vision for how teachers and students can work together to create arguments. The purpose of the vignettes is to help you develop your own vision of teaching for argumentation, as well as provide concrete examples of the teaching moves you will need to support argumentation in your classroom.

The lead vignettes in Chapters 2, 3, 4, and 7 tell the story of Ms. Cooper and her students working on a series of lessons that lead them through the stages of argumentation about coordinate geometry. Other vignettes portray single lessons that address one or more parts of argumentation. We see both types of lessons as important—those that fit together to cover all the parts of argumentation and those that focus on one part of argumentation. Both types of lessons must address important content goals, and the lessons in our vignettes do so.

Following the vignettes, Chapters 2 through 5 and 7 on the parts of the model each contain four more sections:

- **Teaching moves** that can be used to elicit cases, conjectures, justifications, or conclusions from students
- **Classroom norms** that are important for each part of argumentation and warm-up games that you can use to help inculcate these norms
- **Planning** for each part of argumentation that will help you think through in advance how to deploy teaching moves and select tasks
- **Mathematical tasks** appropriate for the middle grades, covering important content aligned with many current state standards. Tasks are accompanied by activity handouts structuring the relevant parts of the model and, where appropriate, interactive web pages created with GeoGebra and Desmos, where students can investigate geometric shapes and algebraic representations.

Chapter 6 presents different levels of sophistication of students' justifications. This chapter is based on classic research in this field and supplies teaching moves for helping students increase the sophistication of their arguments.

Chapter 8 provides tools and a process for planning argumentation-rich lessons for use with your students. A *planning form* guides you in thinking about which particular moves may be useful in that lesson, as well as possible student responses. Our *visualization technique* helps you turn those written plans into rich imaginings of what may happen that will guide you when you actually teach a lesson.

Every chapter ends with a Working Together section with an agenda for working with other teachers in a professional learning community to implement the advice given in each chapter.

Reaching the full diversity of students is central in our work. In Access for All sections, found in shaded boxes throughout the book and marked with an icon, we highlight how our methods address the needs of students who have been historically marginalized in mathematics—students of color, students with learning differences, and students who are English Language Learners (ELLs). Often, digital tools provide means for access for all, and there is a section on digital tools in almost every chapter.

ACCESS
FOR ALL



HOW YOU CAN USE THIS BOOK

You don't have to read the whole book to get started. In Chapter 1, we offer four steps for getting started with argumentation that you can begin tomorrow. You can use justification in almost any mathematics lesson you normally teach, as you get students to discuss the reasons behind the answers they give to problems. Then, as you get further into the book, you can focus on, for example, promoting bold conjectures as you read the chapter on conjecturing. Each chapter contains advice you can implement as you gain confidence in teaching for argumentation. If you are already integrating argumentation into your mathematics discourse, you can choose chapters on areas in which you want additional help.

HOW WE DEVELOPED THE APPROACH IN THIS BOOK

The advice in this book was derived from our work over the past 8 years on Bridging Professional Development, a project funded by the National Science Foundation and conducted through SRI International. Bridging helps teachers learn specific teaching moves for supporting mathematical argumentation through specially designed curricula and through the use of teaching games based on improvisational theater games. These games also inspire activities for establishing productive norms in the classroom—warm-up games—as we describe across several chapters in the book.

In three Bridging projects, we worked with teachers in a diversity of districts through several iterations of our professional development program. Much of our program design and approach to argumentation is rooted in on our team's collective experiences in the classroom and as teacher professional developers. We also drew from the best research on argumentation done over the past 3 decades. We are particularly indebted, for example, to the work of Imre Lakatos for sparking some of our initial thinking about the nature of mathematical argumentation. Through each iteration of our program, we worked closely with teachers in co-design activities and other rich interactions to refine our approach and

explore in depth the kinds of supports teachers would really need to broaden access for a diversity of student populations to mathematical argumentation.

We also had promising results in two small-scale studies with teachers participating in Bridging programs. In one study, we found that providing teachers with curriculum activities focused on conjecturing and justifying, along with training to use teaching moves to support argumentation, enabled them to facilitate argumentation talk in their classrooms. In another study, we provided teachers with a 2-week unit focused on learning geometry through argumentation, and we found that students demonstrated learning gains from pretest to posttest. While these findings are promising in general, you will need to use your own judgment as a practitioner to decide how to apply Bridging's approach in your own unique setting. You can read more about the details of our professional development program and research in these additional publications: Knudsen, Lara-Meloy, Stevens, and Rutstein, 2014; Knudsen and Shechtman, 2017; Knudsen, Shechtman, Lara-Meloy, Stevens, and Rutstein, 2015; and Shechtman and Knudsen, 2011.

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