



REASONS TO REASON IN PRIMARY MATHS AND SCIENCE

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REASONS TO REASON IN PRIMARY MATHS AND SCIENCE

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1

A RATIONALE FOR REASONING

IN THIS CHAPTER

By the end of this chapter you will:

- have considered definitions of thinking and reasoning;
- understand how thinking and reasoning are similar yet different;
- be able to begin to identify **skills of reasoning**;
- have reflected upon the place of reasoning in at least two primary STEM subjects.

Introduction

This chapter will introduce you to a number of important ideas around how primary-aged learners can, do and could reason within their mathematics and science education. As well as explanations and illustrations the chapter will give examples and opportunities to reflect and reason yourself.

The ability to question, pose, investigate and solve problems is at the heart of mathematics and science. However, until a problem has first been understood, learners may struggle to engage with it. This presents learners with many difficulties, because there is a great deal involved in solving a problem; a fact that is often unappreciated. There are many skills involved in unravelling what a problem is about, which concepts and skills are needed and how to make use of them in finding a possible solution. Some of the required knowledge and skills will be subject-specific, some will be common to mathematics and science – for example, measurement.

Even when learners are equipped with several different investigative and problem-solving strategies they still need to reason which one to use. Reasoning is essential to children learning more and

solving problems. It is the one element that binds together all the different skills needed to solve a problem (skills such as pattern spotting, offering conjectures, generalising). Without reasoning, learners may be simply following procedures, applying rules, ignoring patterns and missing opportunities. While this book is about reasoning, we acknowledge that this is one of several strands that learners need to draw on to be mathematically or scientifically proficient. Kilpatrick et al. (2001) offer five different strands: conceptual understanding, procedural fluency, strategic competence, productive disposition and adaptive reasoning (see Chapter 7). They view these strands as equally important and interdependent. Being able to draw on each of these stands when solving problems and carrying out investigations is key to being successful in these two STEM subjects – and without the reasoning strand, learners may struggle to make progress, connect ideas and knowledge and reach a level of understanding that is not simply about producing a formula or memorising a fact. So, what is reasoning?

What is reasoning?

The word ‘thinking’ is used in everyday language and we use it in different ways. For example, *Anne is sure to think of a solution* (an action); *What do you think about the decision to cut down these trees to enable a housing development?* (seeking an opinion); *Why didn’t you think of that before you went ahead with that idea?* (a kind of foresight).

Types of thinking have been described by different people. We do not have space here to examine these in detail but Frank Williams’ (1969) taxonomy lists eight types of creative thinking:

- fluency:** the capacity to generate ideas, possible responses to a problem or situation
- flexibility:** coming up with alternatives, different ideas
- originality:** generation of new unique solutions
- elaboration:** the development or expansion of ideas to make them more comprehensible/interesting
- risk-taking:** experimenting, trialling, challenging ideas
- complexity:** applying logic, establishing order, identify missing parts
- curiosity:** wondering, puzzling
- imagination:** the ability to see a mental picture, new ideas, new possibilities

It is interesting to note that reasoning is not explicitly mentioned in Williams’ list.

In talking with teachers and reflecting about thinking and reasoning, we have settled on a definition that, for us, is a point of reference which we can draw on when exploring reasoning in primary classrooms:

*Reasoning is thinking,
but it is thinking in a logical, purposeful, goal-directed way.*

We have selected the words carefully within our definition. To apply some logic assumes that alternative viewpoints have been considered; to have purpose implies that we are focused and determined in pursuing our line of enquiry; and to have a goal suggests that there is an objective in mind, but this does have to be a final answer or a proof – this could be the process we have been engaged with or the knowledge we have gained along the way. We purposefully did not include the word ‘correct’ in our definition. This is because, for us, reasoning is how we navigate through a problem, to try and make sense of the situation, to select the tools to help us and to reach some sort of resolution, even if this is to continue to reason! We also believe that all learners can reason, despite researchers such as Inhelder and Piaget (1958) suggesting that learners’ ability to reason is quite limited until around the age of 12 years old.

So, for us, reasoning is another type of thinking, some may even say a subset of thinking. In some cases it is about finding reasons for things, making decisions, considering cause and effect, wondering, What is the reason for this happening? While reasoning is not mentioned in Williams’ (1969) list of thinking, it would contribute to all his types of thinking if we applied our definition to it.

How do reasoning and thinking fit together?

Thinking is that exercise of the mind which forms ideas, conceptions, conditions, recognitions, revisions, considerations, connections, opinions, judgements, imaginings and intentions. Since our brains are so very active it is perhaps no surprise that we have so many terms associated with thinking. Reasoning is a form of thinking; words such as logical and systematic often preface. But thinking can be disordered, illogical and messy. Reasoning is, according to Fowler and Fowler (1984), *the action of thinking in a logical, sensible way even if the outcome is incorrect*. Reasoning brings with it a certain order (often subjective and pertinent to that person); it suggests clarity and logic.

REFLECTION

When do you reason in your day-to-day life? When making a purchase? Choosing what to eat? Which route to take to work?

The terms ‘reasoning’ and ‘thinking’ are often used interchangeably. On one level this is understandable and acceptable, but on another, it de-values the importance of reasoning and the skills associated with it.

To assist your understanding, take a moment to look at the options below. For each, consider does this require thought? Does it require the logical, purposeful, goal-orientated thought we call reasoning?

Do these situations require reasoning?

- (a) Choosing a birthday present for a friend.
- (b) Deciding how to vote in a referendum.

- (c) Selecting a new pair of shoes.
- (d) Deciding how to pay for the purchase of a house.

Each scenario requires thought. Options a) and c) might, in some circumstances, require little thought or in others more than that. Options b) and d), it might be argued, require more thought and the weighing of options and consideration of rationale we associate with reasoning. Equally, different people will employ different **skills of reasoning** for each scenario. For some, buying a pair of shoes is a fairly straightforward experience which may require some reasoning, but for others it may require many more reasoning skills.

By now you are probably thinking I need one definition for thinking and one for reasoning. You are reasoning about reasoning!

What do others say about thinking and reasoning?

There are many varied and interesting definitions of thinking and reasoning.

Thinking is using thought or rational judgment while reasoning is to persuade, or move by argument, to express in logical form.

(Fowler and Fowler, 1984)

Mathematical thinking is more than being able to do arithmetic or solve algebra problems. In fact, it is possible to think like a mathematician and do fairly poorly when it comes to balancing your checkbook. Mathematical thinking is a whole way of looking at things, of stripping them down to their numerical, structural, or logical essentials, and of analyzing the underlying patterns. Moreover, it involves adopting the identity of a mathematical thinker.

(Devlin, 1991)

Probably the single most important lesson is that being stuck is an honorable state and an essential part of improving thinking.

(Mason et al., 2010)

Reasoning is fundamental to knowing and doing mathematics. Reasoning enables children to make use of all their other mathematical skills and so reasoning could be thought of as the 'glue' which helps mathematics make sense.

(Pennant et al., 2014)

Adaptive reasoning refers to the capacity to think logically about the relationships among concepts and situations. Such reasoning is correct and valid, stems from careful consideration of alternatives, and includes knowledge of how to justify the conclusions.

(Kilpatrick et al., 2001)

In considering the different definitions of thinking and reasoning we asked teachers what their views were. We have included one example to show how we collected these definitions, and a summary (Figure 1.1).

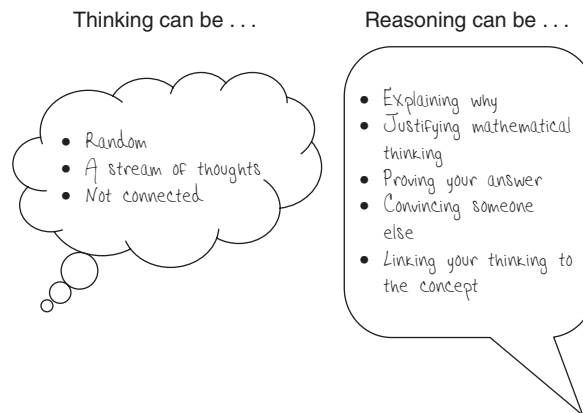


Figure 1.1 An example of the differences between thinking and reasoning given by a group of teachers

Table 1.1 Definitions of thinking and reasoning

What is thinking?	What is reasoning?
Thinking can be recalling facts.	Reasoning is a form of thinking.
Thinking can be exploring, pondering.	Reasoning is purposeful, a logical progression.
Thinking can be open-ended, abstract, no particular end point.	Reasoning is trying to make sense of things; it is deeper than thinking.
Thinking can be everyday thoughts, uncoordinated.	Reasoning is about logic, deeper thinking, convincing, proving, explaining.
Thinking can be a cognitive process that doesn't always culminate in a product.	Reasoning is the why and the how.
Thinking can be low level recognition or consideration of ideas.	Reasoning often uses more than one idea or concept to amend, develop or establish concept.

The teachers we consulted all perceived reasoning to be a higher order skill than thinking. They believed that reasoning required thinking but you could think without reasoning.

REFLECTION

What do you think? Is reasoning a higher order skill than thinking?

However, many of the teachers also commented that they had not really considered the difference before we asked them to, and confessed they were probably using the words interchangeably. While this is not a problem within everyday language and conversation, within an educational context there is a difference, because, as we show in Chapter 2, reasoning has many

important and necessary skills which mathematics and science draw heavily on, that are different to reasoning, in that they are logical, purposeful and goal-directed.

TRY THIS!

Have a ten-minute discussion with colleagues to consider the difference between thinking and reasoning. Collate their responses on a flip chart and display it in the staff room for a few weeks/half a term. Invite teachers and teaching staff to reflect on and add to the list.

Historical perspectives of reasoning

Reasoning has a history, perhaps going right back to the time when our ancestors became aware of themselves and developed an ability to think. Did they reason that life in a dry cave was more comfortable than outside? Did they reason that piles of stones could be arranged with wooden timbers to make a human-made cave? A shelter, a dry, potentially warm and safe place to live?

Ancient Greece is seen as the birthplace of modern thoughtful endeavour including philosophy, mathematics, logic and scientific method. Socrates (470–399 BC) was famous for a form of discussion or argumentation about ideas, the Socratic method. Plato (428–348 BC) wrote of the value of abstract ideas. He saw them as more powerful than observation of reality. Their contemporary, Aristotle (384–322 BC), formed the basis of much Western philosophy and he had a profound effect on science following his study of and writing about topics including optics, motion and biology. These and other thinkers (Abu Nasr Al-Farabi, 872–950 AD) showed that rational thought could take humans forward. Such thinking could seek explanation and extrapolate from one observation or idea to another. Many early ideas were later proved wrong, but crucially they were thoughts, they were rational and they empowered others ultimately to challenge these ideas and seek better alternatives.

Why do we need reasoning?

Why do people need to reason?

Humans may be the only sentient organisms which reason. Our ability to reason allows us to develop ideas and artefacts which benefit ourselves and the world. Could reasoning be the currency of the future? We use reasoning in games, in our work, in pastimes and in relationships. In fact, games like chess are often promoted as opportunities to develop reasoning. To gain an understanding of learners' reasoning we often use theoretical models (e.g., Piaget, Vygotsky, Bruner). These theories are themselves sets of ideas where we and others reason about their potential.

Why do we need reasoning in mathematics and science?

Mathematicians and scientists need to reason. They need to observe and recognise features and explanations relating to values, relationships, patterns and phenomena. Science includes facts to be

learned. Mathematics is a subject based on rules that need to be explored. Without reasoning, much of mathematics and science would be under-developed and perhaps would remain unknown to us – for example, Galileo’s reasoning that the four objects he observed early in 1610 moving close to Jupiter were not stars but Jupiter’s moons and that Jupiter and its moons were a model of our solar system (a sun orbited by planets and moons) (White, 2007).

Why do we need reasoning in mathematics and science education?

Both of these subjects run the risk of being taught in an abstract, procedural way. Reasoning can prevent this by encouraging learners to notice, reflect, question and explore. If we do not include reasoning within mathematics and science curricula we are not enabling learners for the future.

Many mathematics educators (e.g., Pennant et al., 2014; Nunes et al., 2009) have considered reasoning to be one of the essential components in mathematics education. However, reasoning often lacks the attention in the curriculum that mathematical content knowledge is given (e.g., learners’ ability to recall number facts). Subject knowledge content is essential to children learning mathematics. But if we do not teach this content alongside reasoning, opportunities to extend learning are missed. Look at the following mathematics example. The calculation could be completed with very little reasoning, but what does the reasoning component add to the questions that follow it?

REASONING FOCUS

Solve this calculation: $14 \times 5 = ?$

Have you reached an answer?

How did you work it out?

Which method or strategy did you use?

Can you solve it a different way? And another? And another?

Reason: Which method is the quickest? Easiest? Most efficient?

Look in the Appendix for some possible solutions.

Science can also be abstract but primary science is usually focused on everyday phenomena which can often, but not always, be observed directly; for example, the insulation of sound, the appearance of shadows, the growth of a seedling. Reasoning assists young scientists with things that puzzle them, such as explanations, links, questioning, problem-solving. Reasoning supports the learner who looks at a situation or a phenomenon such as a rainbow and applies thought. They might utilise (see Chapter 2) one or more of the six powerful words: Why? When? How? Where? Who? What? For example, Why does a rainbow form? When does a rainbow form? How does a rainbow form? Where does a rainbow form? Who might contribute to answers or investigations? What causes a rainbow to form?

Primary mathematics and science are often concerned with questions, predictions, data and patterns. Looking at the graph in Figure 1.2, can we extrapolate from the data presented to suggest a figure for the world population in 2051? Could we reason and consider factors which might influence how steep the line will be? Food supply? Health? Birth rate?

Cross and Borthwick (2016) quote Haylock (2010) and The Royal Society (2014) when they argue strongly for links between primary school mathematics and science. Connecting mathematics with science makes sense because the skills strongly overlap between the two STEM subjects. Often when we talk about mathematics and science we reference the specific subject knowledge content (such as fractions in mathematics or plants in science). However, we need to remember that while both subjects have prescribed content knowledge, the **skills of reasoning** are applicable to both.

We have already suggested that questioning is imperative to reasoning in mathematics and science education. Consider these six sentence stems and how they engender a reasoning atmosphere: Why?, When?, How?, Where?, Who?, What?

Mathematics example – Does the multiplication of two or more numbers always result in a larger number?

Why? Why do you think multiplication results in a larger product?

When? When you multiply numbers together, what is happening mathematically?

How? How could we multiply numbers together?

Where? Where can I find an example that offers a counter example?

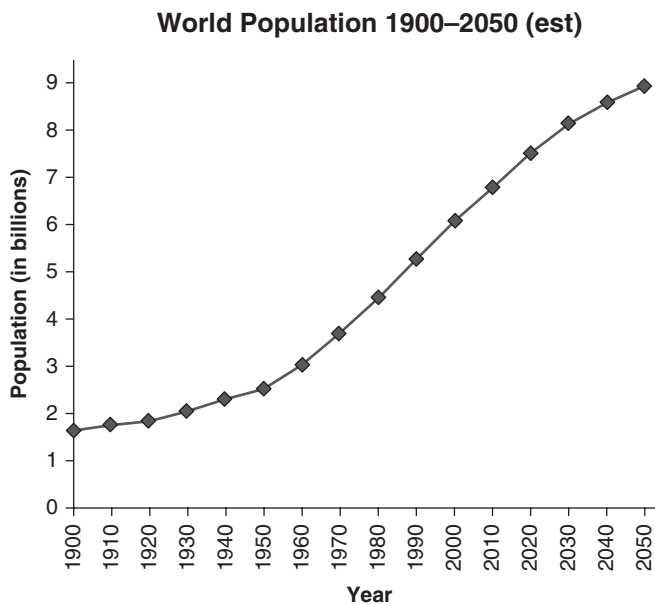


Figure 1.2 Estimated growth in the world population of humans on Earth

Who? Who could prove or disprove this question?

What? What do I need to do first?

Science example – Can sound travel around corners?

Why? Why does it travel like this?

When? When can sound travel?

How? How do we know how sound travels?

Where? Where does sound travel?

Who? Who can tell us more?

What? What do I need to do to find out more?

Without reasoning in mathematics and science education, learning would be very limited.

TRY THIS!

Go on a reasoning walk around the school. What evidence can you spot of learners' and adults' reasoning?

How do we reason?

There are a number of stages that learners might go through when they are reasoning. These are related to the skills we explore in Chapter 2 which looks at **skills of reasoning**. *How* we reason is a process we go through; often with stages intertwined with each other and not always in the order below. Look at the suggestions below.

- noticing;
- talking and listening;
- making sense;
- sharing insights;
- metacognition.

The importance of noticing

If we do not notice something, how can we begin to reason about it? To notice something means we have made some observations – perhaps we have been surprised or ambushed in some way, but

it has caused us to question, stop, pause, reflect, think and then we begin to reason. We believe noticing is particularly important in reasoning.

The importance of talking, listening, sense-making and sharing insights

Talking and listening are particularly important for learner reasoning. Primary teachers will generally agree that learner talk and discussion in lessons is very important. We want to emphasise here how very important talk is in the development of reasoning in both science and mathematics.

Reasoning is about thinking and making sense of the world. Sense-making is something that individuals do as they assimilate ideas and accommodate (Piaget, 1970) new ideas in relation to those previously held. Vygotsky (1978) talked of each learner's internal world of thought 'inner speech' which then develops through our making thoughts external (e.g., sharing them).

One way we do this is through Barnes' (1992) 'exploratory talk' in social settings, in this case, classrooms. The zone of proximal development (Vygotsky, 1978) describes the area where a child learns in the presence of a knowing other, adult or child. Communication, much of it verbal, will be through talk. Therefore Mercer's (2008) objective of 'fluent and reflective speakers' is not only the objective of education but also the means to learn, developing through its expression.

This has an advantage for teachers because in order for us to assess understanding prior to teaching we need to perceive children's thought. Our only way of doing this is through verbal interaction alongside observation of behaviour and of children's written work. Learners need to interact, to share insights, but much more important than that for reasoning is the idea of 'inter-thinking' – that is, thinking with others (Mercer and Littlejohn, 2007).

For class teachers two important related ideas can assist us: these are collaborative learning and one means to harness this, dialogic teaching (Alexander, 2017a). Again, collaborative learning is more than learners simply tackling tasks together; it needs to be taught, for example, developing interpersonal skills. Collaborative learning can be articulated as the co-construction (Reznitskaya, 2012) of ideas. Individuals working together to, for example, clarify, apply, justify ideas and understanding. These words describe dialogue, which is much more than conversation.

Compare these short extracts, do they show thought? Reasoning?

Child 1 – My favourite number is 7.

Child 2 – I like them all.

Child 3 – 7's mine, too.

Child A – The biggest number of all is 100.

Child B – How do you know?

Child A – It's bigger than 99.

Child B – What's $99 + 10$?

Child A – ... Oh ... that's bigger! It's 109.

The second extract shows how the learners' thinking has developed through the use of the question, How do you know? This question caused a chain of reasoning, which drew on some explanation, justification and refuting. The first extract could have easily led to some reasoning, but it stopped with the stating of a fact and the discussion around this.

One proven approach which aims to raise the quality of thinking through dialogue is dialogic teaching (Alexander, 2017a, 2017b). This idea recognises different kinds of talking but that two of them, discussion and dialogue, are less common in classrooms. Dialogic teaching supports learners and devotes time to having learners express, talk about and discuss ideas, then justify them, speculate and more, for example, making links and connections. One approach might be Mercer, Dawes and Staarman's (2009) talking points, where learners discuss ideas that are factually accurate, contentious or downright wrong.

REASONING FOCUS

Would these talking points lead to discussion?

Human pollution is bad for all life on Earth.

People cause all pollution.

Poorer countries should be allowed to pollute until they get richer.

People should not pollute the natural world.

Electric cars don't pollute.

You might also be familiar with *Concept Cartoons* (Naylor and Keogh, 2010) which also enable discussion.

REFLECTION

Language skills are essential to reasoning in mathematics and science, but they are important to all subjects. Can we apply these ideas across the curriculum? Can all subjects contribute to reasoning? In this way could the effect become cumulative?

The importance of metacognition

All thinking skills, including those we would recognise as reasoning, involve a degree of self-regulation. As this develops it contributes to self-awareness and thinking for oneself. We would advocate encouraging learners to think about their thinking, what Flavell (1979) called 'metacognition', for example,

to reflect on how they learn best. While the skills outlined in this book begin with activities that support the teacher to help, model, question and direct learners, it is important to allow learners to develop their skills of reasoning independently, and without over scaffolding from the teacher. In fact, an over eagerness to support learners with reasoning may actually limit their opportunities to reason! One of the purposes of increasing learners' capacity to reason is for them to find structure or impose meaning on the problem they are trying to solve. Learners' reasoning will improve when they are allowed to think about their thoughts; we might encourage them to be self-reflective thinkers.

TRY THIS!

In lesson plenaries ask learners to reflect.

What have I learned? How will this help me in the real world? Can I use this in other subjects? Do I now think about things differently?

What do teachers think?

Before we examine where reasoning fits into the current English National Curriculum (DfE, 2014) you might first want to ask yourself if *you* think it does. We surveyed 44 primary teachers (Cross and Borthwick, 2017) about reasoning in the primary subjects. For them, mathematics – and to a slightly lesser extent, science – were the two subjects with the greatest potential for developing reasoning. Three other subjects (English, design and technology (DT) and history) were also selected by around one-third of the teachers. Other subjects such as PE, geography and religious education were identified by around one-quarter of the teachers. The 44 teachers appeared to have an understanding of the need for reasoning and some of the benefits it offered. One-quarter of them felt that reasoning is an innate skill, half felt it can be learned and one-quarter felt it was both innate and learned. Almost all were quite clear that primary-aged learners could reason and that reasoning began with preschool-aged children. When asked to select features of classrooms which enabled reasoning, about half the teachers selected open-ended questioning, child-led discussion and exploratory activity with around one-quarter selecting child-led activity, encouragement and acceptance of non-conventional answers. The options of closed questions and teacher-directed activity were each selected only once.

TRY THIS!

Why not survey your teachers/staff in school?

You can find the survey we used in Chapter 9.

When asked about reasoning itself, the teachers talked about thinking, deduction and problem-solving. They said that they value reasoning and feel that they can recognise it when they see it.

Almost all felt that professional teachers reason a great deal. But only a handful felt it was straightforward to teach. They could suggest activities that would lead to learners reasoning and could identify opportunities for learners to reason. For some, reasoning was a form of directed thinking or thinking about a problem. They often talked about everyday examples of reasoning, for example, reviewing a diet.

We also asked a group of 38 student teachers (PGCE) to draw learners in a lesson where their activity had led to the children reasoning (Figure 1.3). Twenty-five of these drawings included mathematics activities, with five featuring science and the remaining eight featuring other primary subjects or general activities. All the drawings were plausible and regularly included learners working in pairs and groups. Other drawings showed learners talking to one another, or in some cases working alone. Speech bubbles suggested that lone learners were talking to themselves – what might be called inner-speech (Vygotsky, 1978). These examples all implied a cognitive engagement with a task, very often a problem-solving task. The illustrated learners were engaged with a task or context, were usually using or observing some form of equipment, had observed a relationship or phenomenon and appeared to be seeking a solution or explanation. There is a limit to what can be communicated in a drawing but the drawing activity and the discussion that followed provided useful reflection and even reasoning about reasoning. Student teachers felt that group work would be advantageous, that lessons should include time for learners to reason and that reasoning would often be stimulated by practical enquiry or engagement with a problem.

Almost all the student teachers found it challenging to suggest activities that would lead to reasoning, however all of them, working in pairs, managed to make several suggestions. They were less forthcoming when asked how to teach reasoning and struggled when asked about how learners' reasoning might develop progressively.

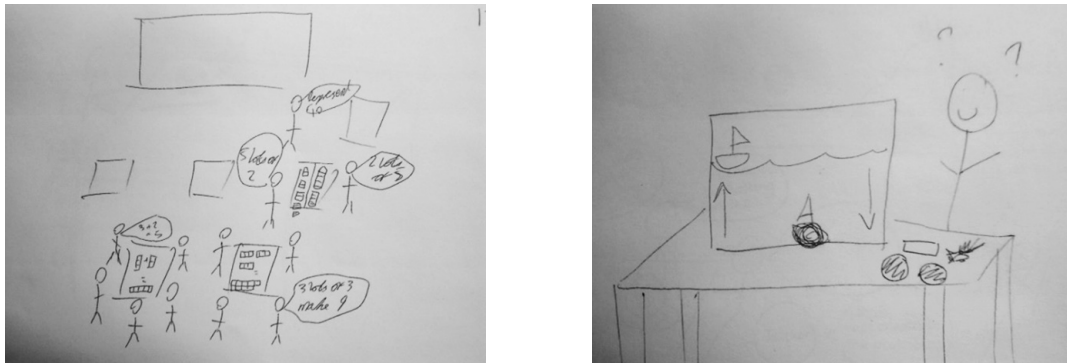


Figure 1.3 Student teachers' drawings of learners engaged in activities that led to reasoning

TRY THIS!

Ask learners or teachers to draw a picture of what it means to reason in mathematics or science.

Both teachers and trainees saw potential in mathematics and science for reasoning and could identify tasks and challenges which would enable reasoning; however, they were less confident when asked to suggest ways to teach reasoning.

REFLECTION

Is this an example of a learner reasoning?

The image shows a learner's handwritten work on grid paper. At the top, the equation $5.40 \div 2 = 2.70$ is written. Below it, a long division calculation is shown: $2 \overline{) 5.40}$. The quotient 2.70 is written above the line, and a checkmark is drawn to the right. Below the division, a subtraction problem is shown: $240 - 4 = 236$, with a horizontal line under the 4.

Where does reasoning fit into the English National Curriculum?

Reasoning is one of the core aims of the latest version of the English National Curriculum for mathematics (DfE, 2014):

The National Curriculum for mathematics aims to ensure that all pupils reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.

(DfE, 2014)

It is also included in the annual National Curriculum tests (STA, 2016a) that all learners aged 11 are required to sit (STA, 2016a). In the 2016 tests the question $5542 \div 17 = 326$ was included. Learners were asked to explain how they could use this answer to work out the following: 326×18 . Only 33 per cent of learners across England were able to answer this correctly and 21 per cent did not even attempt the question.

Could it be that too much emphasis is placed on practising abstract questions which rely heavily on mathematical content, rather than teaching the **skills of reasoning**? Perhaps many learners looked at this question and assumed they had to calculate either a difficult

division question, or a long multiplication question, rather than reasoning they could add 326 onto 5542?

Using reasoning skills to solve this question (e.g., pattern spotting, altering, making links) would have potentially resulted in more learners getting this question right than the 33 per cent who did. While learners need to show competencies in solving tricky calculations, they are rarely going to use them in real life (mobile phones usually include a calculator as one of the free applications). For this question, using reasoning skills would have made the question much quicker and easier to solve. Perhaps then, one suggestion as to why we should reason and teach the **skills of reasoning** is that if we do not, we are making the mathematics potentially more difficult than it needs to be.

While reasoning is one of the three aims, alongside fluency and problem-solving, in the English National Curriculum for mathematics, there are no specific objectives to support the teaching of reasoning within the yearly programmes of study. This makes it tricky for teachers to understand what reasoning looks like in each year group and within each mathematical area, let alone remember to include it. As we have already suggested earlier in this chapter, reasoning is key to the solving of problems. It is a skill which is essential to learners progressing with their mathematics, and so while it may not be as obvious as some of the more conventional mathematics we find in the curriculum (such as the learning of number facts or addition of fractions), it is arguably more important than the content provided.

In the English National Curriculum for science (DfE, 2014) there is no specific reference to reasoning, though there is mention of deeper understanding, abstract ideas and that science ideas change and develop over time. The non-statutory guidance contains references to forms of thinking, including analysing, exploring ideas, comparing and contrasting, researching, considering patterns, etc. While the term 'reasoning' is not used, it is clear that reasoning would enable the National Curriculum objectives and in particular what it calls working scientifically (DfE, 2014).

Learners appear able to gain knowledge in science, a strong feature of the English National Curriculum for science (DfE, 2014), but appear less able to use this knowledge. In a recent sample of testing in England learners were asked about candles burning in upturned beakers. After establishing that candles in different beakers are extinguished in different times, 11-year-olds were asked about a beaker twice the size and the effect this would have. Seventy-six per cent of learners gave a reasonable response, for example, doubling the time based on the information given. The remaining learners, while realising that the candle would burn for longer, made insufficient use of the information they had to deduce a reasonable conclusion (STA, 2016b).

Look at the example in Figure 1.4. What are you thinking? Are you reasoning?

As humans, most of us are full of ideas. Sometimes when there are a multitude of ideas it is hard to hold onto them and make sense of them. Using reasoning skills helps us to order these ideas, make sense of them and see the next steps in acting on them. When we reason we can use physical models, diagrams and research but we always have to engage with a context which often employs a question.

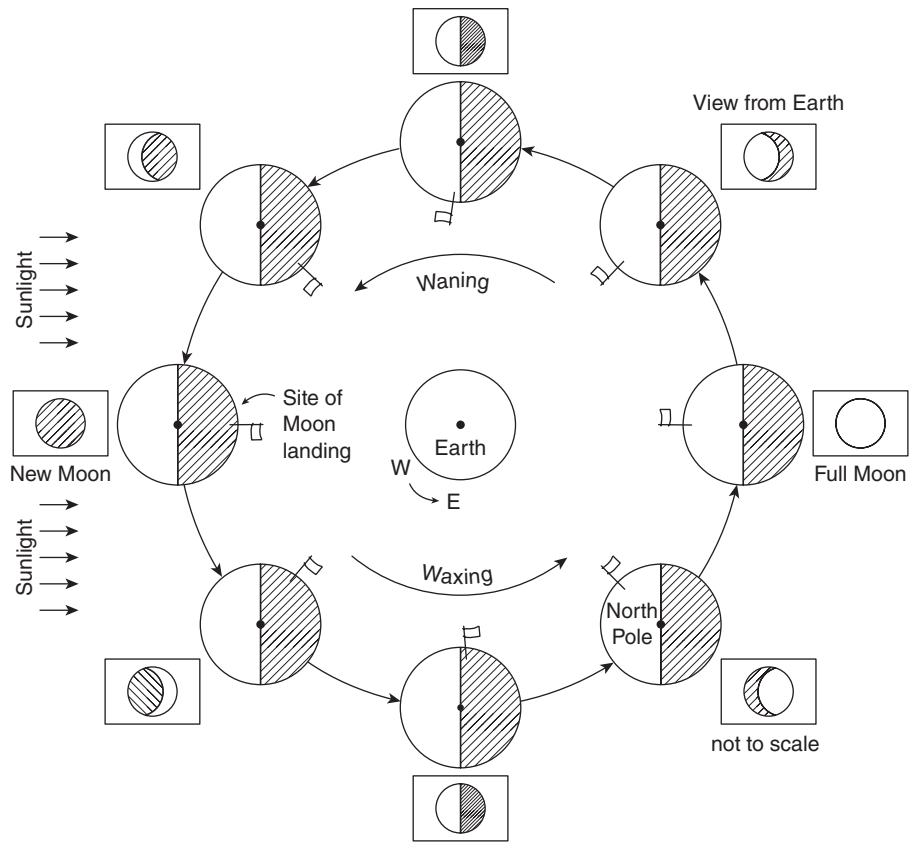


Figure 1.4 One side of the Moon always faces Earth, the other side gets equal periods of light and dark

REASONING FOCUS

The so-called 'dark side of the Moon' is never visible from Earth - so if you lived there would you ever see the Sun in the sky?

Find the solution in the Appendix.

Table 1.2 Chain of reasoning

Chain of reasoning	
About what is called the 'dark side of the Moon'	
I know the Moon orbits Earth.	Something I know.
I've read that the Moon spins (slowly in comparison to Earth).	A new idea to me.

I know only one specific half of the Moon ever faces Earth.	I remember this and suspect it may be linked to the answer.
Does that mean that all the Moon's surface experiences periods of light and dark?	I have a question which I think may help me.
So therefore, does the side which never faces Earth get as much light and dark as the rest of the Moon?	So this is another question.
So is this just the side we never see from Earth? It is just called the dark side but in truth it experiences 50% light and 50% dark just like the rest of the Moon's surface?	Does this question and this conjecture help me?

Would you recognise this as a chain of reasoned thoughts? It involves: recognising what I know; something I have noticed; something else I know which may be linked; another question; and a conjecture. Chapter 2 explores **skills of reasoning** further.

So, how might we teach learners to reason?

If we are going to teach reasoning, we first need to be able to recognise when learners appear to be reasoning. We asked teachers to imagine they were looking into a classroom. How would they know if the learners are reasoning? What would they be looking for or noticing?

Teachers said that learners might be:

- engaged in animated discussion;
- talking to each other;
- talking to the teacher;
- having a debate;
- disagreeing about something;
- using equipment;
- not writing in their books;
- appearing thoughtful.

Teachers said that learners would not be:

- working in silence;
- recording individually;
- disengaged;
- simply chatting;
- working passively.

REFLECTION

Do you agree/disagree with the above?

Could learners be working in silence yet still reasoning?

How would you know?

We then asked the teachers, 'What would the teacher have needed to do to enable the learners to reason?'

The teachers said that teachers might have:

- given the learners a hook to spark their interest;
- been quiet, listening to the learners;
- posed a misconception or a mistake (perhaps it is written on the board?);
- organised the classroom furniture in a certain way (e.g., learners are sitting in a group or in a pair, or perhaps they are lying on the floor);
- provided learners with large sheets of paper to encourage group recording;
- asked the learners a question;
- encouraged questions from learners;
- begun a chain of reasoning and asked learners about the next step.

REFLECTION

Do you use any of the above?

What impact do they have on learners' reasoning?

In writing this book we felt it was important to seek opinions from teachers and learners. Chapters 2 to 9 outline what we think are the skills of reasoning and offer ways to include these within lessons.

REFLECTION

What is your maths/science capital?

What maths/science do you know?

What do you know about teaching maths/science?

What is your attitude to maths/science?

What is your personal interest in maths/science?

What are your personal experiences of maths/science (in any form)?

Who do you know who knows, values, uses maths/science?

CHAPTER SUMMARY

This chapter has considered the reasoning we use in our everyday lives and how reasoning contributes to and is contributed to by primary mathematics and science. It has defined both thinking and reasoning and emphasised the importance of reasoning and the skills associated with reasoning. The chapter will have given you background and a flavour of what is to come in the book including ways we can reason and ways in which we can extend learners' thought with reasoning.

Having read this chapter you will:

- have considered definitions of thinking and reasoning;
- understand how thinking and reasoning are similar, yet different;
- be able to begin to identify **skills of reasoning**;
- have reflected upon the place of reasoning in at least two primary STEM subjects.